The Third Step: Parallactic Shift of Venus I : *Concepts*

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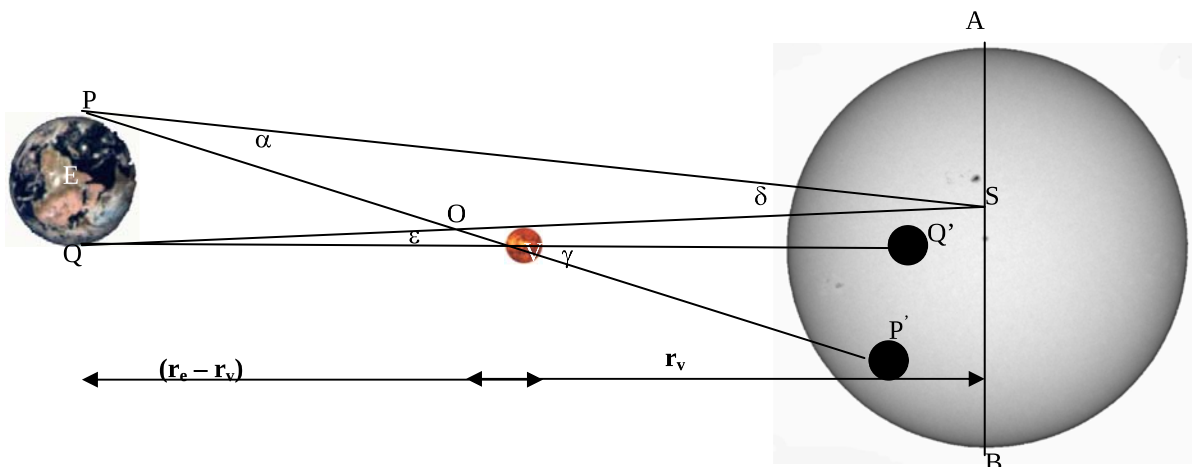
Background

Parallax refers to the apparent shifting of an object when viewed from different angles. If you view the same object from two different positions, the perspective will change. Hold your thumb out in front of you at arm's length and view it first with your right eye (left eye closed). Note the background. Then with your left eye (right eye closed) repeat the experiment. Your thumb appears to jump! Actually it is the background that changes. This shift is called a **parallactic shift.** The **distance** from the bridge of your nose to the thumb in your extended hand can be calculated by combining this **shift** and the distance between your two eyes (called **base line**) with a little geometry**.** Notice that this shift is estimated by noting the change in the fixed background. It is this simple principle that will be used on the transit day.

Let us start with some definitions. The **parallax** π_s **of** the sun is defined as the subtended by the radius of the earth R_e at the sun.

$$\pi_s = R_e / d_s$$

In the figure below E, V and S are the Earth, Venus and the Sun. Observers at positions P and Q will see Venus at P' and Q' on the solar disc. Here positions P and Q correspond to the two eyes. **PQ is the base line**. Venus is the thumb and the solar disc is the background. $P'Q'$ **is the parallactic shift** of Venus. If we can measure the base line PQ and the angle $\angle P'VQ'$ we can calculate the distance between the Earth and Venus EV. But the distance to the sun ES, which forms the background, is only about 3 times larger so the solar disc itself has a parallactic shift when viewed from locations P and Q. That is the background is not a **fixed** background! To find the true parallactic shift of Venus, the parallactic shift of the **sun itself** has to be added to the observed parallactic shift. But we are in fact interested in measuring the parallax of the sun. During the transit, the earth rotates on its axis and revolves around the sun. Venus also is in orbit around the sun. It is as if the base line changes orientation and the reference object it self is moving! All this changes the viewing geometry from instant to instant. Let us see how we can interpret the geometry of the transit to actually obtain the parallax of the sun.



Assume that P,Q and VENUS are in the same plane. In the figure given above the chord PQ subtends the $\angle PSQ = \delta$ at the sun. This angle is related to the parallax of the sun through the equation

$$\pi_s = \delta R_e / PQ$$

Consider the triangle POS: Let $\angle OPS = \alpha$; exterior angle $\angle SOP' = \alpha + \delta$.

It is also the exterior angle in triangle QOV. Therefore $\angle SOP' = \gamma + \varepsilon$

$$\alpha - \varepsilon = \gamma - \delta$$

$$\alpha = \angle P'PS = P'S\,\rho/AB; \; \varepsilon = \angle Q'QS = Q'S\rho/AB.$$

where $P'Q'$ is the linear distance measured on the image of the solar disc and AB is the diameter of the solar disc measured in the same units and ρ is the angular diameter of the sun as seen from the earth. Let $P'S/AB = f_p$ and $Q'S/AB = f_q$ then $\alpha = f_p\rho$ and $\varepsilon = f_q\rho$. $\alpha - \varepsilon = \rho\,(f_p - f_q)$, the parallactic shift of Venus that can be measured.

Since γ and δ are small angles subtended by the chord PQ at Venus and the sun respectively

$$\gamma/\delta = r_e/(r_e - r_v)\,;(\gamma - \delta) = \delta r_e/(r_e - r_v) - \delta = \delta/(r_e/r_v - 1) \text{ Thus}$$

$$\alpha - \varepsilon = \rho\,(f_p - f_q) = \gamma - \delta = \delta/(r_e/r_v - 1)$$

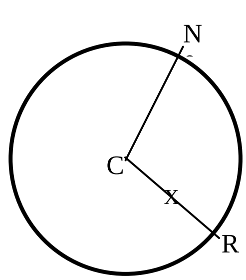
$$\delta = (r_e/r_v - 1)\text{x}\,\rho\,(f_p - f_q)$$

We have already measured the ratio r_v/r_e in Phase II of the Vamana Project. The other quantities in the right hand side will be measured on June 8, 2004. From δ, π_s the parallax of sun can be calculated from $\pi_s = \delta R_e/PQ$.

If we plug in the known values for, the angular diameter of the sun, the ratio of the radii and radius of the earth to be the chord, we can calculate the parallactic shift of Venus as a fraction of the image diameter. Such a calculation yields the result that the shift is just~3.4% of the diameter of the sun even for two stations separated by the diameter of the earth. The parallactic shift that will be seen on the solar disc is barely equal to the diameter of the shadow of Venus on the disc! Several techniques have been developed to measure this **small** shift as accurately as possible. Remember that for an arbitrary combination of observing stations P and Q the above treatment will be a 2-D approximation. Again the base line PQ has to be measured perpendicular to the Sun earth line. So the actual base line that must be used in the calculation is the projection of the chord PQ on this line. The angle of projection can be calculated from the latitude and longitude of the two locations. In the March 21 experiment of the Vamana Project the latitude and longitude of many locations have already been determined. So you see the subtlety of the calculation increases many fold as we take all the factors into consideration!

Measuring the Parallactic shift- The method of Chords

If two observers measure the position of Venus as seen on the solar disc at the SAME instant then a comparison of the two positions will give the parallactic shift of Venus. It is easy to see that BOTH positions must be measured in the same co-ordinate system established on the solar disc. Let the centre C of the solar disc be the origin of this system. Consider a point X on the solar disc. Draw a radius CR passing through X. The radial coordinate of X is CX/CR where CR is the radius of the solar image. Both CR and CX are measured in the same units,

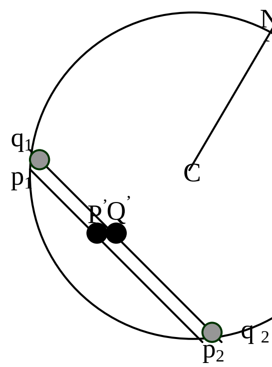


Let CN be the direction of the earth's polar axis projected on the solar disc. The second coordinate of the point X is the angle <NCX measured counter clockwise. This is called the **position angle PA** of the point X. The PA of X = $360^0 - \angle NCX$. Thus the coordinate system

1. Centre of the solar disc as origin
2. Projection of the north pole on the disc as the reference radius.

So both observers have to measure the position of Venus observed from their locations with reference to the centre of the solar disc and the radial projection of the earth's axis of rotation on the disc. It may not be possible for the two locations to have observations at the same instant. If observations are made at two different instants from the same site, the chord in which Venus appears to travel across the sun can be established. The figure below shows the composite of two diagrams made from two different locations, scaled to the same disc size and the poles aligned. The separation between the chords gives the parallactic shift. This method is not of a very high accuracy but is simplest to understand.

Halleys Method



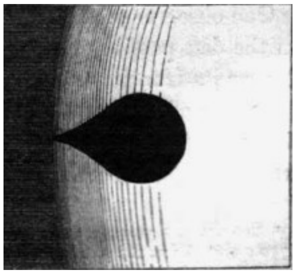
Because of the difference in perspective, the observer at P will see Venus travel along the chord p_1p_2 and the observer at Q will see it travel along chord q_1q_2. As we have already seen the separation of the two chords measures the parallactic shift. But this measurement is not an accurate method for finding the parallax of the sun. Halley suggested that at each location, the length of the chord itself can be measured using the the time difference between the time Venus fully enters the sun (**second contact**, grey circle at q_1) and just begins to leave the sun (**third contact**, grey circle at q_2). Halley realised that between second and third conact, the earth itself would have rotated and earth and Venus would have also have moved in relation to the sun. Therefore the obsereved parallactic shift is a combination of
1. The distance between the two locations (base line)
2. The change in the orientation of the base line due to rotation.
3. The relative motion between Venus and the earth

Both factors at 2 and 3 can also be calculated from the time difference if a minimum of two stations participate in the experiment. Halley's method is superior to the method of measuring the parallactic shift described above as it uses only **differences in times** of contact. Since timing can be done accurately, Halley felt that this method can be very accurate.This method is simple as far as the observations are concerend and has the following important advantages.
1. Timing can be made very accurately
2. As only time differences are involved timing coincidence is NOT required. The differences in the scale of the image do not enter into the process of comparison.
3. Appropriate correction for the rotaton of the earth and relative motion of Venus can also be calculated using the difference in contact times

As in the previous method atleast two widely separated observing locations are needed. In fact if there ae are more than two stations with the similar quality of data then they can all be combined to get a more accurate result

Black Drop Effect

The attempt to use the Halleyes method of chords was first made by international teams that dotted the globe in 1761. What they found was that timing the second and third contacts exactly, was not easy as the image of Venus distored to the shape of a drop and timings varied as much as 20 second and not ¼ of a second as anticipated! This distortion is the famous **black drop effect**.(fig below)



We now know that this is due to a combination of factors: the non-uniform background provided by the sun at the limb, the presence of an atmosphere on Venus and the turbulence in the earth's at mosphere.

These two methods are the simplest of all available methods and a large number of amateur astronomers will use this mehtod on June 8,2004 for measuring the parallactic shift. So ways to minimise the timing error must be found. Before concluding let us list the observations needed for measuring the parallax of the sun in the Vamana Project.

1. Latitude and longitude of location — *Completed in many locations on March 21*
2. Radius of the earth — *Completed on Feb 27,28 and 29*
3. Ratio of the orbital radius of Venus and Earth — *Completed between Feb 20 and April 20*
4. Angular Diameter of the sun — *To be completed in May*
5. Parallactic shift of Venus — *To be measured on June 9*
6. Angular Velocity of Venus during Transit — *To be measured on June 9 (for Halleys method only)*

In the Vamana Project observations for method of Chords as well as Halleys method will be made. We will deal with the practical aspects in the second part of the Third Step.

References

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